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Class of 2023
Mathematics B.S. & Physics Minor
Can the Shape of Our Universe Explain the Dark Matter?
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The purpose of this research was to investigate the nature of dark matter and dark energy using higher dimensional analysis. The five-dimensional formalism of the universe, incorporated with stereographic coordinates, might mathematically encompass the cosmological constant Λ (dark energy) and gravitationally interacting forces (dark matter).

According to the known laws of gravity, the velocities of stars rotating about the center of a galaxy should decrease with distance from the center. However, the observed velocities increase with distance, approaching an approximately constant value (Rich, 2001). The simplest explanation of this phenomenon is the existence of some unknown matter, called dark matter, which interacts with ordinary matter only gravitationally. In contrast to dark matter, dark energy describes the force that is driving the universe's accelerating expansion. Its existence is postulated because, according to Einstein's theory of gravity, the universe should be expanding but its expansion should be slowing down. Tangible matter only occupies 5 percent of the energy budget of the universe, while the cosmological constant (dark energy) makes up approximately 70 percent and the other 25 percent is owing to cold dark matter (Ferreira, 2019). Although there might be possibilities that the universe has a positive or negative curvature on a large scale (larger than the Hubble distance), it is reasonable to assume that the space is perfectly flat for practical purposes (Ryden, 2020). A flat spacetime could be mathematically viewed as an abstract, four-dimensional plane in a five-dimensional pseudo-Euclidean space (Lord, 1976). The results may contribute to the unifying theory of dark matter and dark energy, and the equations that shape our worldview might be modified.

The five-dimensional formalism and the coordinate relationships were constructed using Euclidean geometry. Applying the stereographic projection to the five-dimensional space resolves dark energy. Matter was then incorporated into the picture while a correction term was derived to modify the current theory of gravity. Phenomenology was used to find the potential correction term, and the derived and phenomenological terms were compared as the result of the research.

In a three-dimensional world, the projection of a three-dimensional sphere on a two-dimensional is referred to as the stereographic projection. The plane on which lies above the north pole of the sphere is the stereographic plane. A point on the sphere has coordinate $x_i, x_N (i = 1, 2; N = 3)$. The corresponding 4 coordinates on the stereographic plane are ξ_i $(i = 1, 2)$. The radius of the sphere is R. The relations between x and ξ are

$$x_i = \frac{\xi_i}{1+\xi^2/4R^2}, \; x_N = R\frac{1-\xi^2/4R^2}{1+\xi^2/4R^2}.$$

The equation of the four-dimensional surface is given by $\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 = R^2$. A transformation of the coordinates and application to the equation of the line element of length on this surface gives $R = \sqrt{3/\Lambda}$. The results indicate that the five-dimensional universe can explain dark energy. When matter is incorporated into the picture,

$$\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_4^2 + \eta_5^2 = R^2(1 - \frac{r_g}{r})$$

describes the Kottler metric as a function of r. Consequently, Therefore, Kottler universe is a four-dimensional surface of five-dimensional deformed sphere in not flat five-dimensional space. Traditionally, the function of rotation velocity in terms of radius is given by

$$V = \sqrt{\frac{GM(r)}{r}},$$

while we added the phenomenological correction term and changed the function into

$$V = \sqrt{\frac{GM(r)}{r} + c^2(\frac{r}{R})^{4/3}}.$$

Further research is still needed to compare the derived term with the observed galaxy rotation curves. Once the correction is finalized, the term described by dark matter might be related to this phenomenological term.

References

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